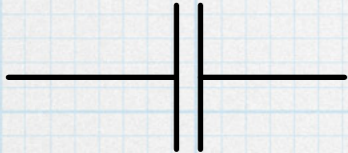

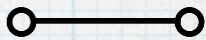

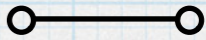



Filter circuits

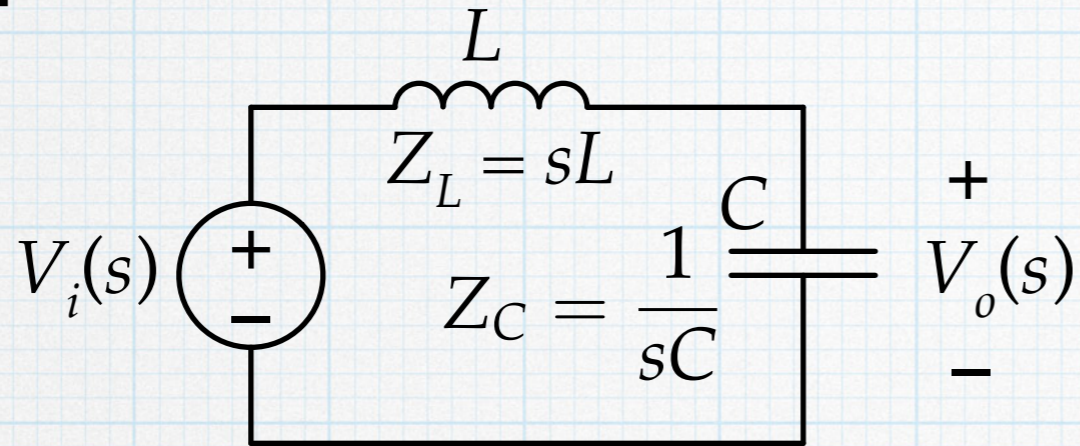
From our work with Laplace transforms (and the AC analysis work from EE 201), it is clear that the behavior of a circuit depends on the frequency used. The impedance of the reactive elements varies with frequency.

	$Z_C = \frac{1}{sC}$	As $s \rightarrow 0$, $Z_C \rightarrow \infty$. An open circuit.	
	$\left(Z_C = \frac{1}{j\omega C} \right)$	As $s \rightarrow \infty$, $Z_C \rightarrow 0$. A short circuit.	

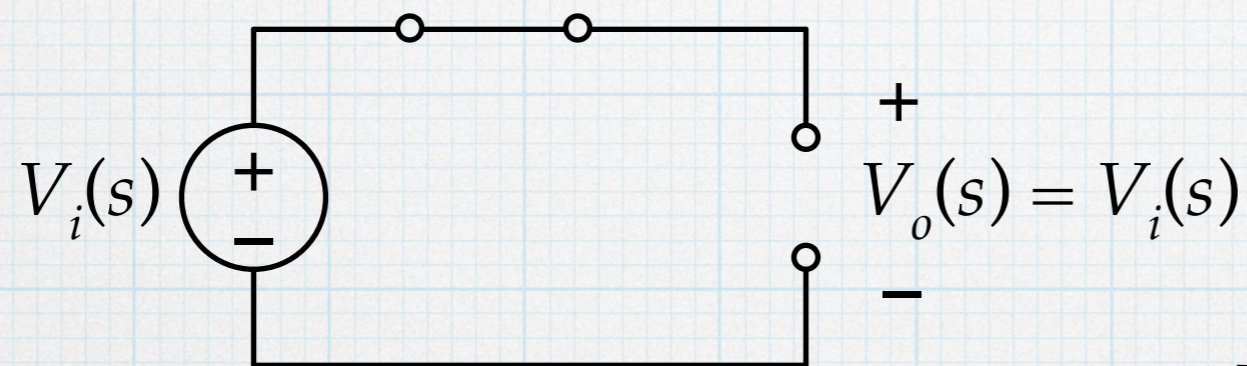
	$Z_L = sL$	As $s \rightarrow 0$, $Z_L \rightarrow 0$. A short circuit.	
	$\left(Z_L = j\omega L \right)$	As $s \rightarrow \infty$, $Z_L \rightarrow \infty$. An open circuit.	

We can use this frequency dependence to build circuits that discriminate between frequencies. The circuit would allow signals at some frequencies to pass through while signals at other frequencies are attenuated – filtered out.

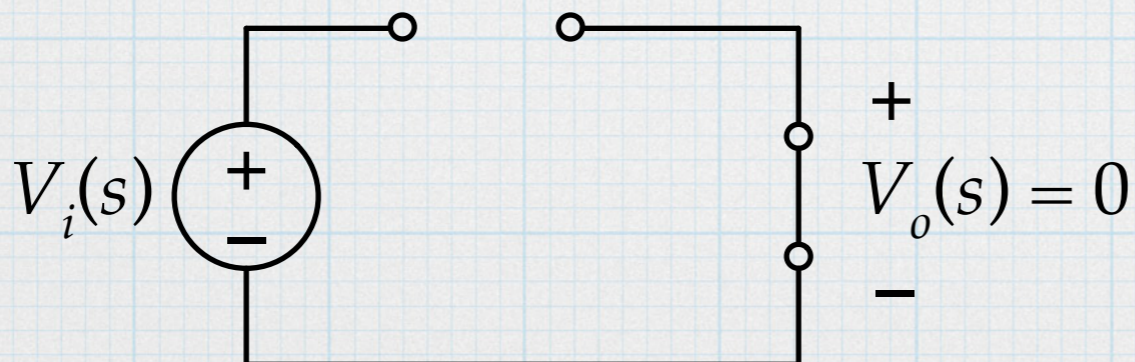
Consider



As $s \rightarrow 0$, $Z_C \rightarrow \infty$ and $Z_L \rightarrow 0$.

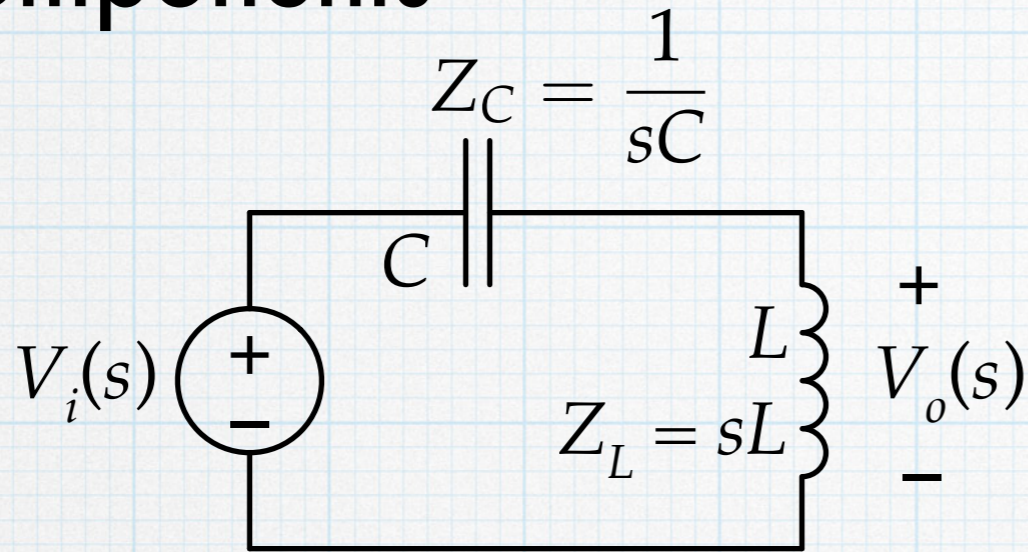


As $s \rightarrow \infty$, $Z_C \rightarrow 0$ and $Z_L \rightarrow \infty$.

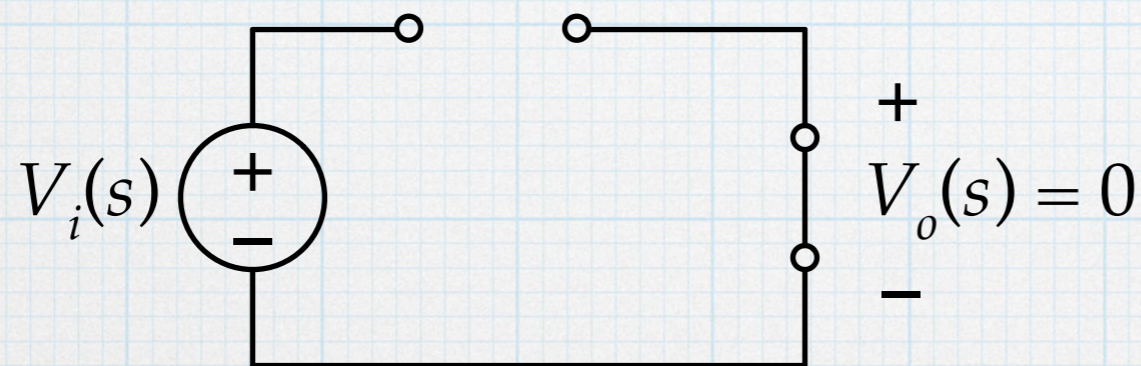


This would be a *low-pass* filter. Low frequency signals are passed through to the output. High-frequency signals are cut off — $V_o = 0$).

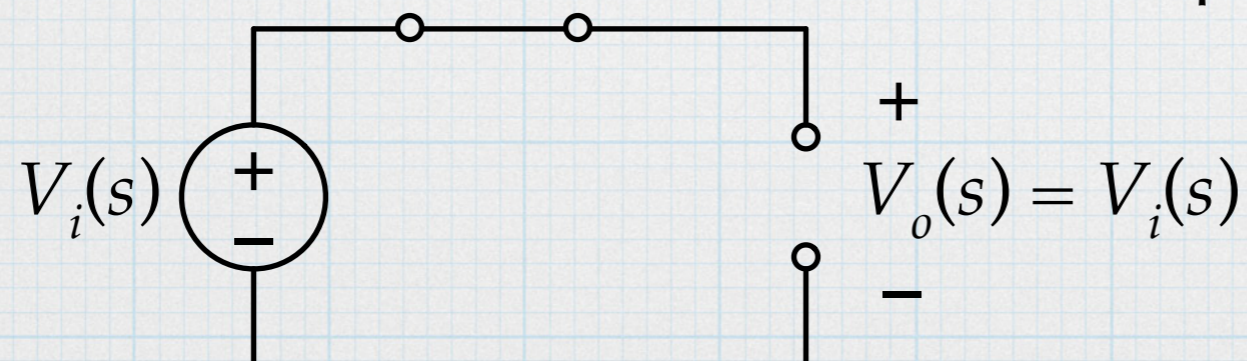
Swap components



As $s \rightarrow 0$, $Z_C \rightarrow \infty$ and $Z_L \rightarrow 0$.



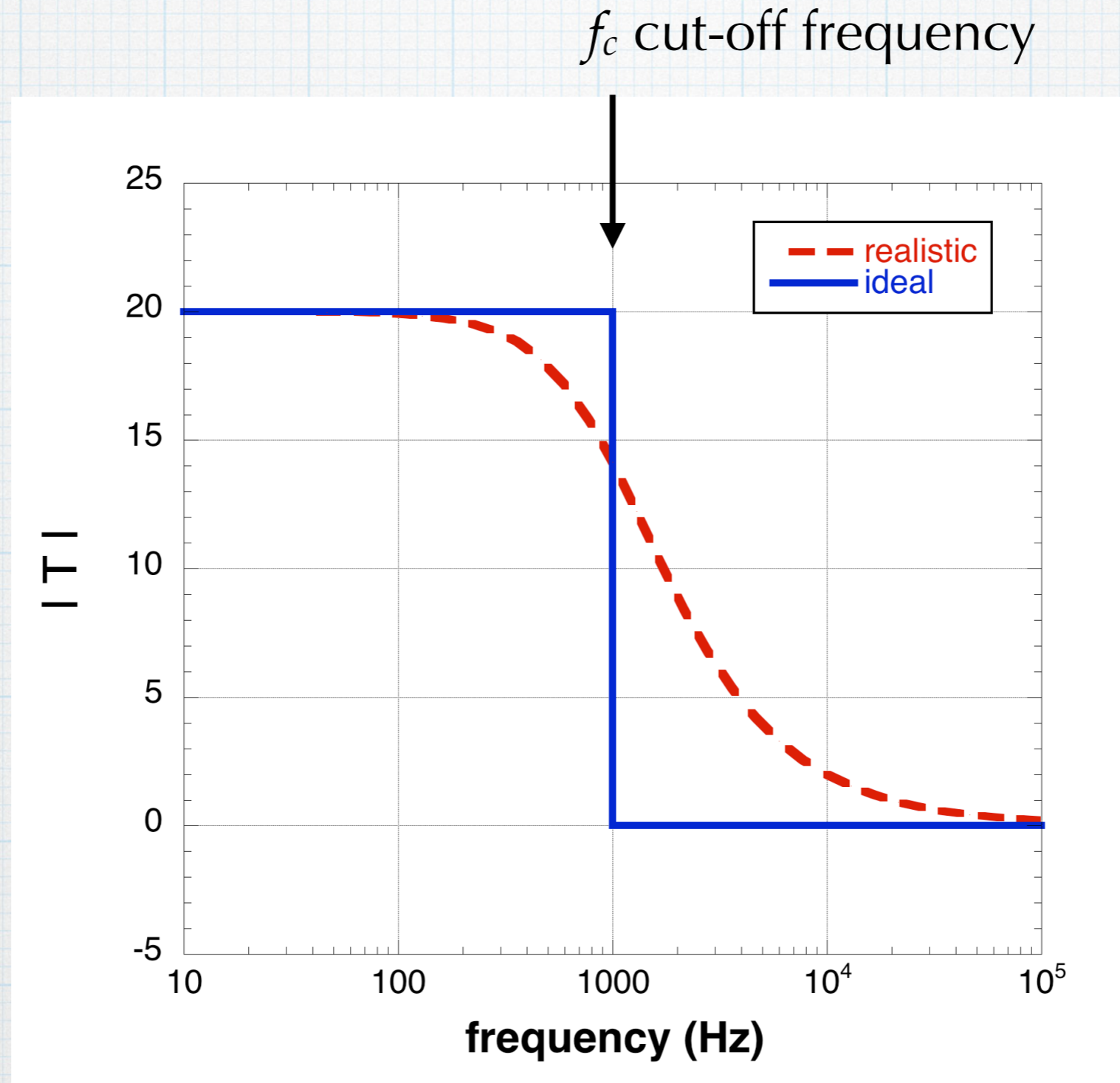
As $s \rightarrow \infty$, $Z_C \rightarrow 0$ and $Z_L \rightarrow \infty$.



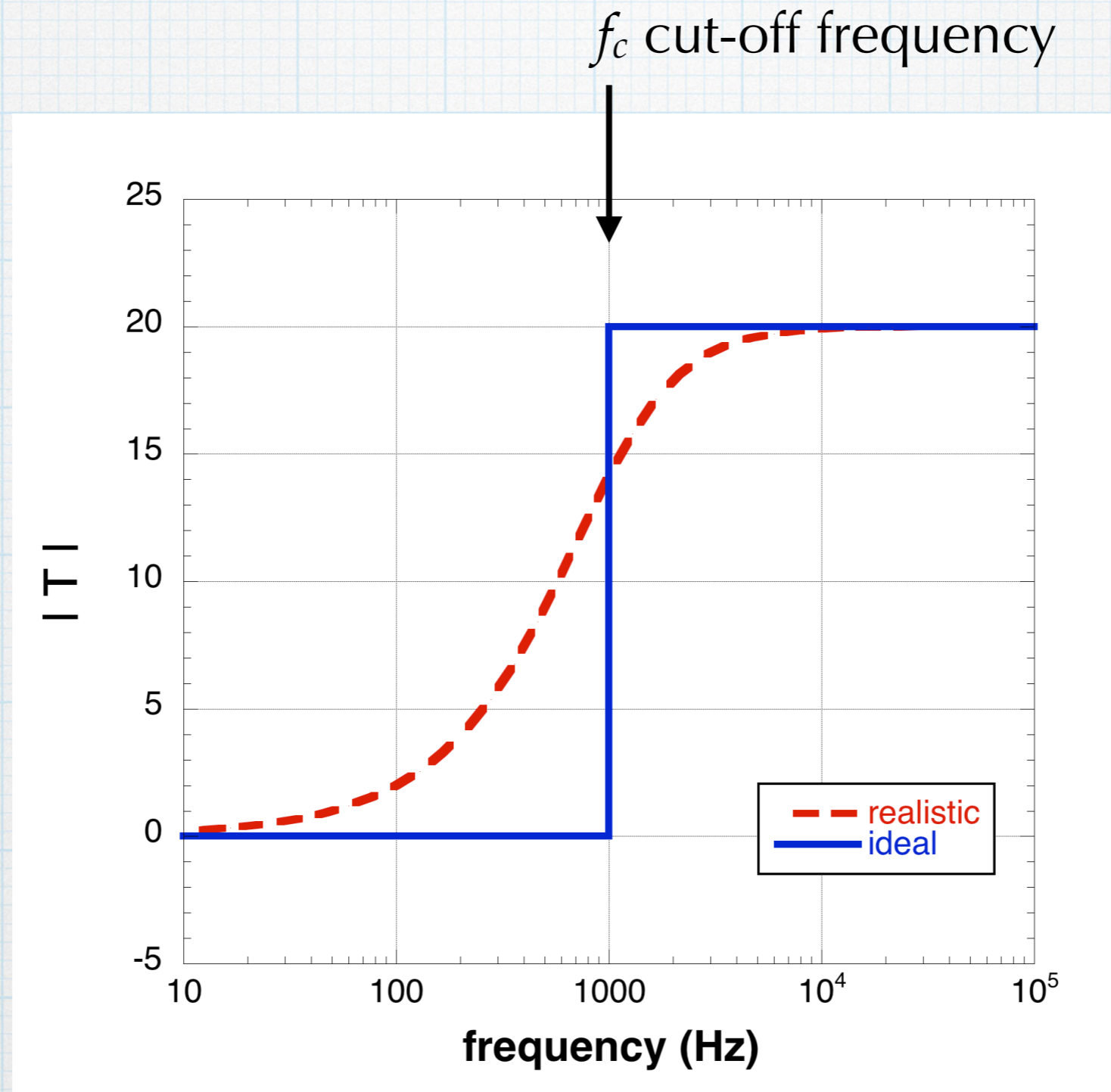
High-pass filter. Low frequency signals are cut off. High-frequency signals are passed through to the output.

Basic filter types

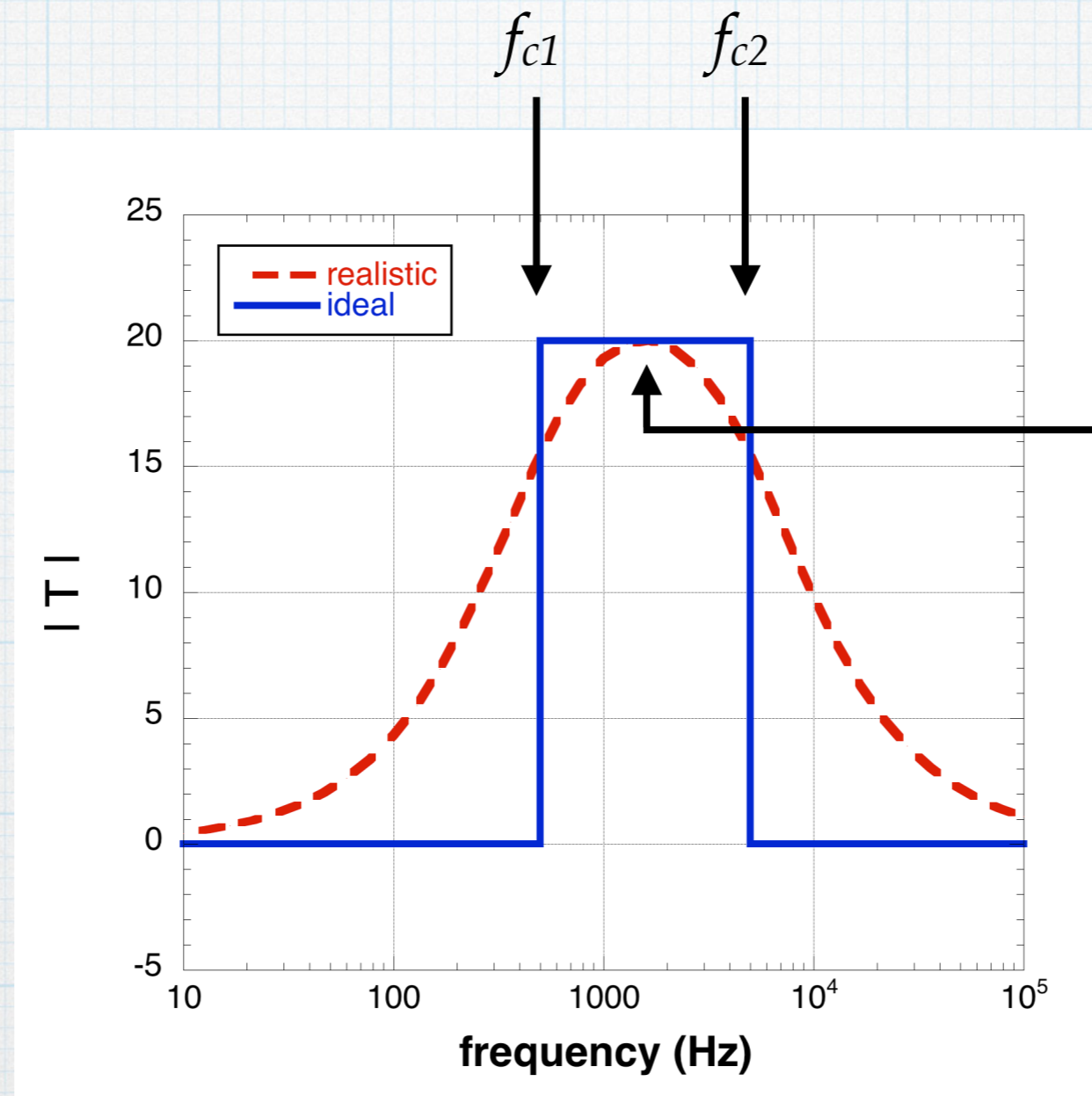
Low Pass: Frequencies below f_c pass through, and those above are cut off.



High Pass: Frequencies above f_c pass through, and those below are cut off.



Band pass: Only frequencies between f_{c1} and f_{c2} pass through. Everything else is cut off.

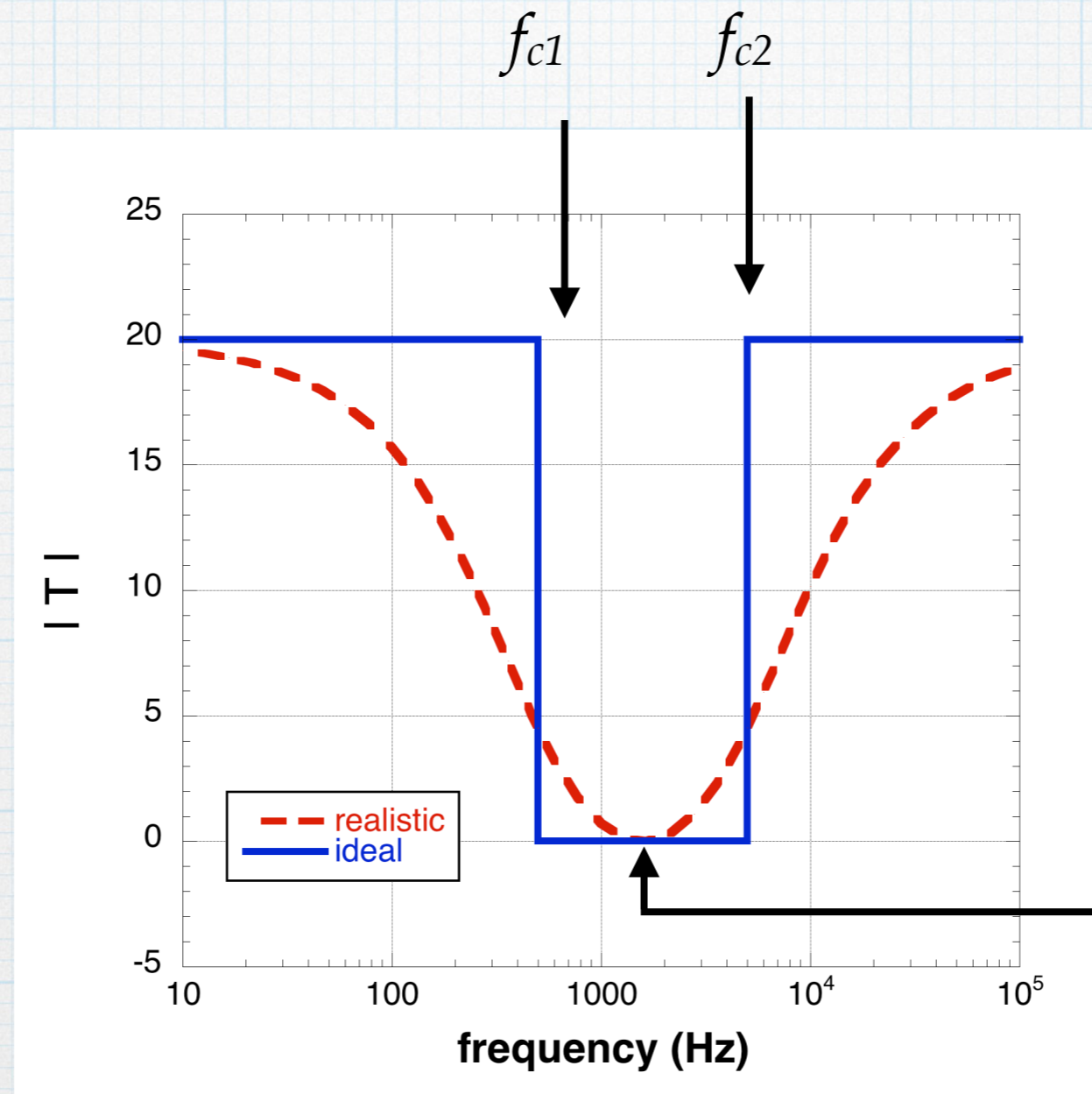


f_{peak} : frequency at which maximum occurs

bandwidth: $BW = f_{c2} - f_{c1} = \Delta f$

quality factor: $Q_P = f_{peak} / \Delta f$

Band reject: Frequencies between f_{c1} and f_{c2} are blocked. Everything else is allowed to pass through.



f_{null} : frequency at which minimum occurs

bandwidth: $BW = f_{c2} - f_{c1} = \Delta f$

quality factor: $Q_P = f_{null} / \Delta f$

Transfer function

The properties of a filter circuit can be specified in detail by the transfer function, defined in terms of either voltages or currents. (Using voltages is most common.)

$$T(s) = \frac{V_o(s)}{V_i(s)}$$

Soon, we will learn how to recognize the basic properties of most filters by examining $T(s)$. However, when we need to compute some specific property of a filter, we will move to AC analysis, where $s = j\omega$.

$$T(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

The transfer function can be plotted as a function of frequency. The graph is called a *frequency response* plot. A complete frequency response is actually two plots, since $T(j\omega)$ is a complex number and complex numbers have two components, described by rectangular (real and imaginary) or polar (magnitude and phase) forms. In working with filters, we almost always use magnitude and phase.

The general shape of the curve is characteristic of the type of filter.

Passive and active filters

Passive filters are made up entirely of passive elements – resistors, capacitors, and inductors. In that case, the power coming out the circuit must be less than the power going in. (Some power may be dissipated within the circuit.)

We can also make active filters, which incorporate one or more op-amps. With an op amp in active filter, the circuit might produce gain, which might be useful in some applications. They also provide for greater design flexibility when making higher order (more complicated) transfer functions.

One downside to active filters is the need for voltage supplies to power the op amps. Also op amps themselves have a limited frequency response and so will not work at arbitrarily high frequencies. We will examine these limits later.

Active filters typically use only resistors and capacitors with the op amps. In principle, inductors could be used as well, but they are physically large and inherently lossy because of the series resistance of the windings. This goes against the mantra of electronics: smaller, faster, cheaper, and lower power. If needed, inductance (positive reactance) can be realized with the right combination of op-amps and capacitors.

Cut-off frequency

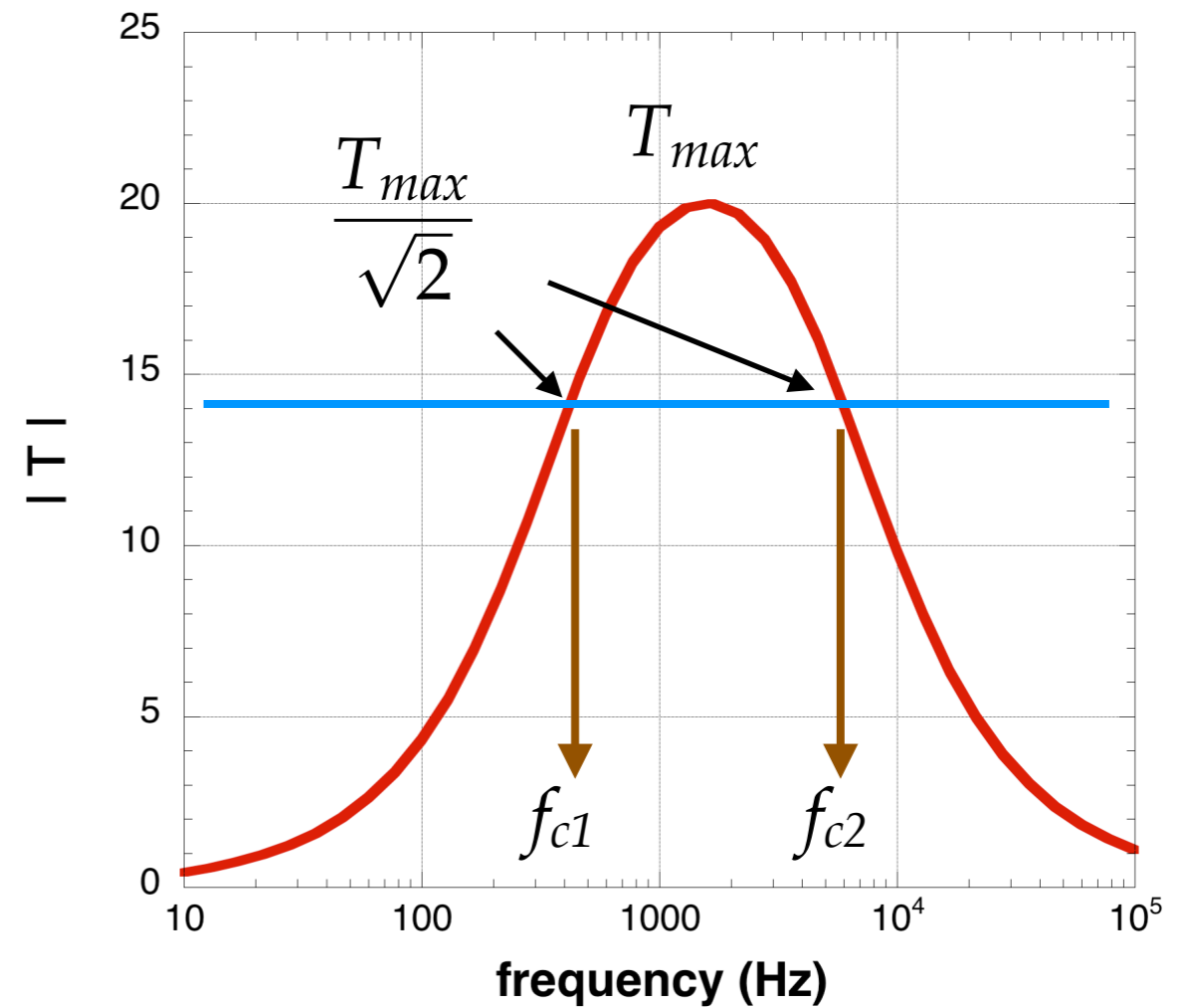
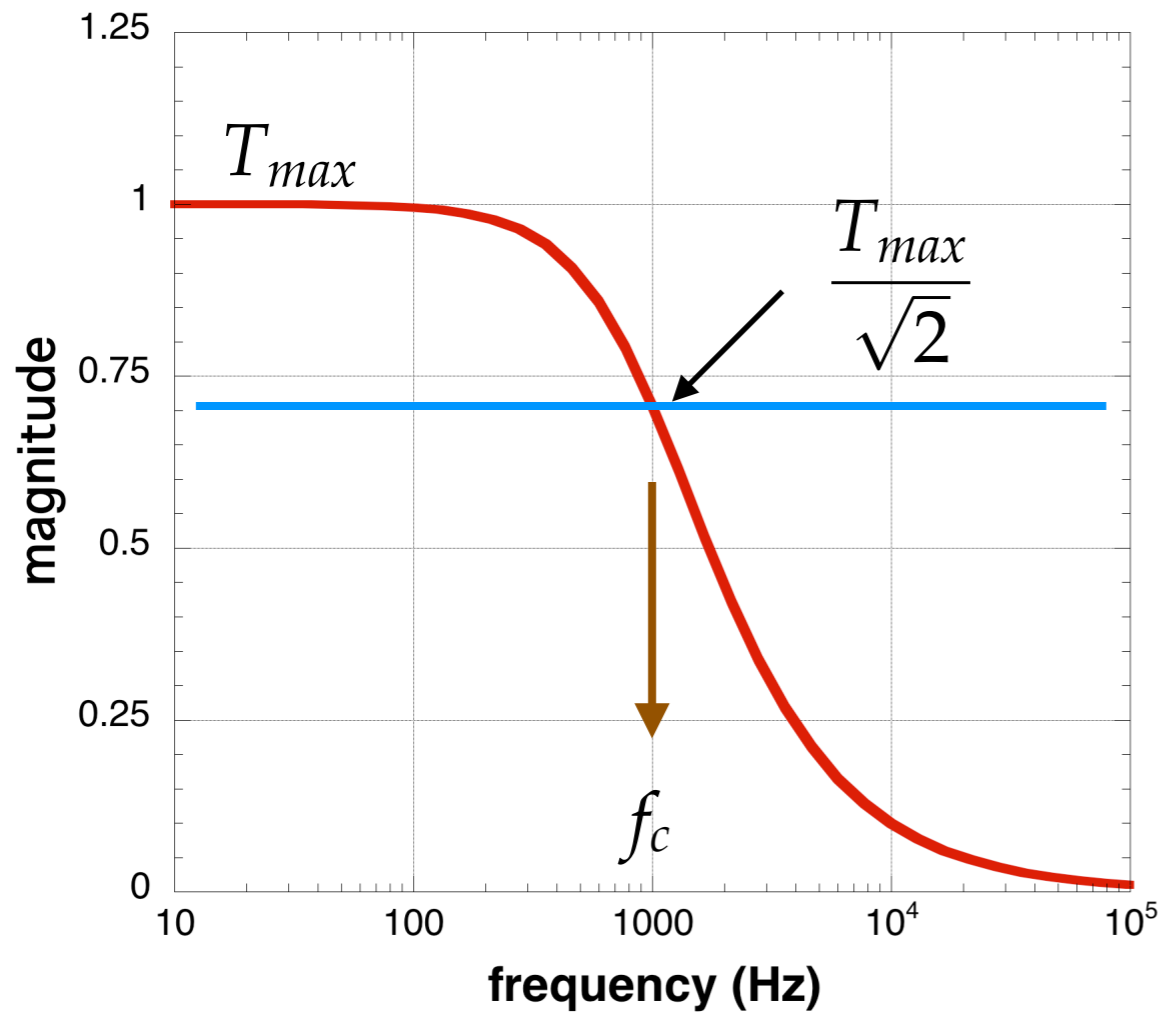
For the ideal filters, it is fairly obvious where to locate the dividing line between the passband and the cut-off region. For realistic filters, the transition is gradual and it is less obvious where the dividing line between pass and cut-off should be.

The usual convention is to choose the frequency where the magnitude of the transfer function is down by $1/\sqrt{2}$ ($= 0.707$) from the peak value.

1. Find the peak of the magnitude of transfer function, $|T(j\omega)|_{max}$ — low frequencies for low pass, high frequencies for high-pass, in the middle of the pass band for bandpass.
2. Find the frequency where the magnitude is $\frac{|T|_{max}}{\sqrt{2}}$

$$|T(j\omega_c)| = \frac{|T|_{max}}{\sqrt{2}}$$

This is also the frequency at which the power being transmitted through a passive circuit is one-half of the power provided by the source. So it is also known as the *half-power frequency*.



f or ω ?

To some degree, it doesn't matter which you use, just make sure that you keep them straight. f is the *real frequency* and is the number of *oscillations* per second (Hz or hertz). This is used in the lab and is usually the number that is reported. The angular frequency ω is the number of *radians* per second. This is what goes into our equations.

Important: $\omega = 2\pi f$!!!!

decibels

As a ratio, the magnitude of the transfer function has no units.

If such a ratio ranges over many orders of magnitude it is sometimes more convenient to talk about the logarithm of the ratio.

magnitude: $1 \rightarrow 10 \rightarrow 100 \rightarrow 1000 \rightarrow 10,000$

logarithm: $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

Using the logarithm, we would describe the ratio in *bels* (after Alexander Graham Bell). The bel is generally a bit coarse for practical use – instead we use *decibels* (dB). 10 decibels = 1 bel

The idea of using logarithms like this comes from early work in quantifying human hearing. The *dynamic range* of the human ear is remarkable. The power of the weakest sound humans can hear is denoted as 0 dB (= 1). Normal conversation is about a million times more powerful (60 dB = 10^6 .) The sound from a jet engine is about 1 million times more powerful than conversation (120 dB = 10^{12}).

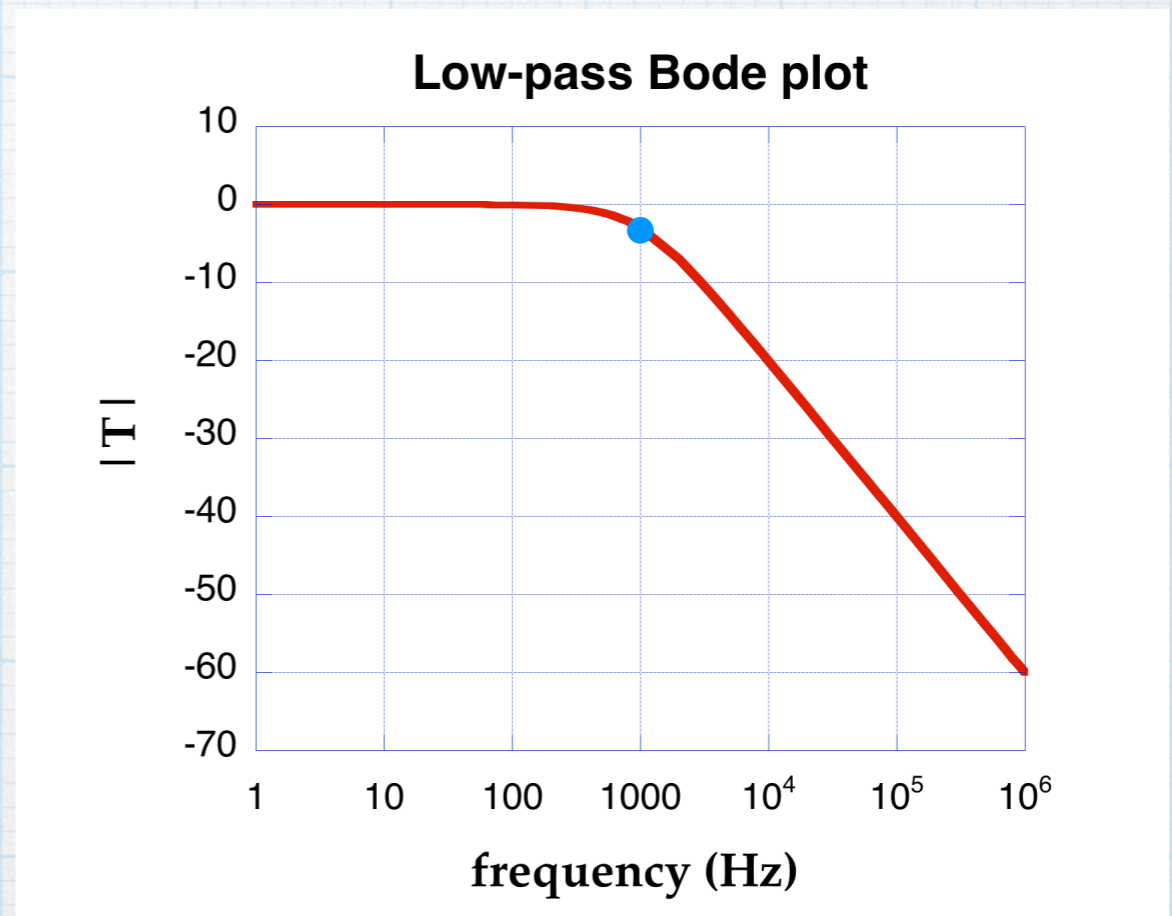
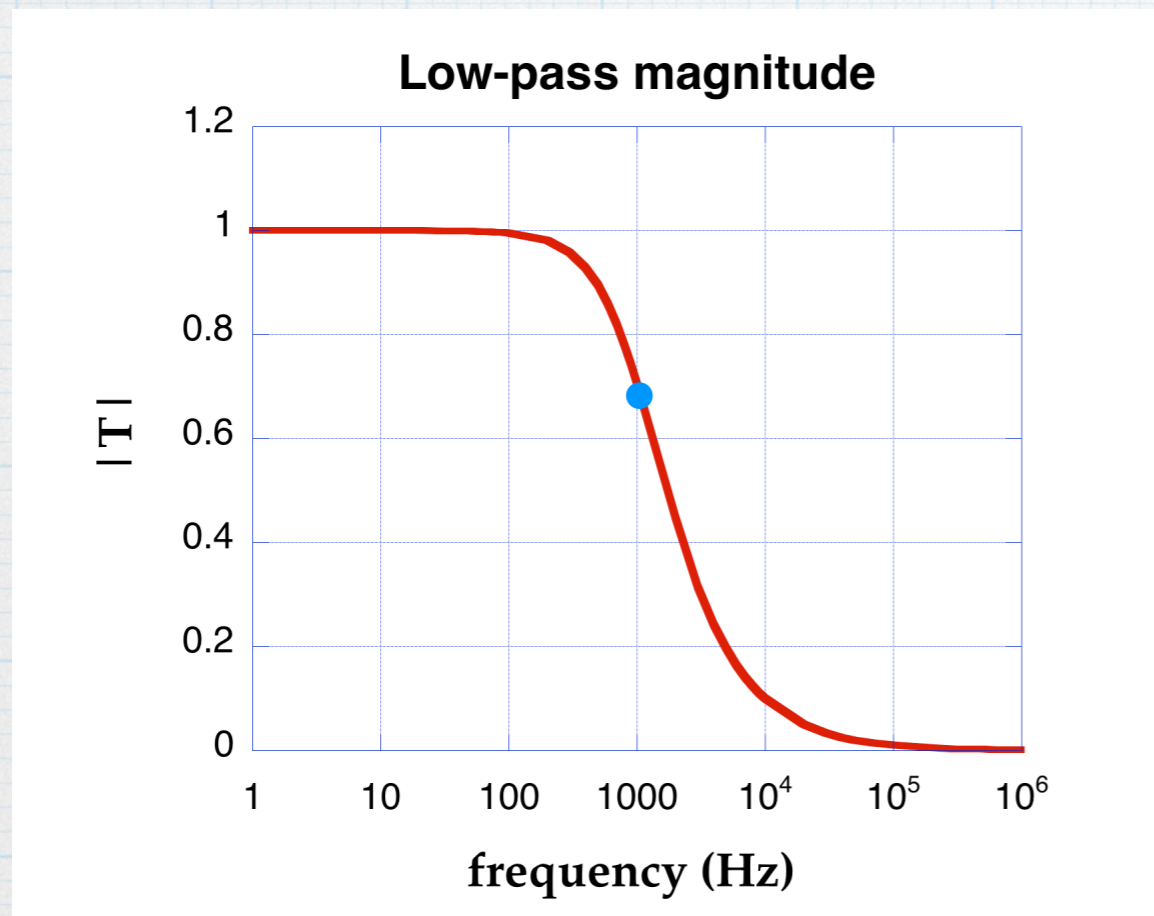
decibel: $0 \rightarrow 10 \rightarrow 20 \rightarrow 30 \rightarrow 40$

Bode plot

In the magnitude plot, using straight magnitude on the vertical axis is OK, but most practitioners make a *Bode* plot, which uses decibels on the vertical axis.

$$f_o = 1000 \text{ Hz}$$

$$\text{decibels} \rightarrow 20 \cdot \log(|T|)$$



$$\text{at } f_o: \quad 20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) = -3\text{dB}$$

corner frequency
cut-off frequency
3-dB frequency